Diffraction Notes

No. 17
Fall 1996

LAMBDA RESEARCH INC. 5521 FAIR LANE CINCINNATI, OH 45227 (513) 561-0883

FINITE ELEMENT CORRECTION FOR STRESS RELAXATION IN COMPLEX GEOMETRIES

INTRODUCTION

Subsurface x-ray diffraction residual stress measurements are necessary to fully assess the magnitude and depth of stress distributions.(1) Layers of material must be removed for subsurface measurement electrolytically to avoid plastic deformation. Re-equilibration after layer removal alters the residual stress in the exposed layers, leading to potential errors in measurement. The resulting errors increase with depth and stress magnitude, and must be corrected in proportion to the relaxation caused by layer removal.

The closed form solutions developed by Moore and Evans(2) allow correction for stress relaxation in simple geometries with uniform stress fields. The method is applicable to subsurface measurements in flat plates where the stress distribution varies only with depth. Hoop or axial stress measurements can be corrected on cylinders or tubes provided the stress field is rotationally symmetric, and varies only with the radial depth. However, no solution has been available for the arbitrary stress fields and complex geometries encountered in components of practical interest, such as gear teeth, turbine blades, bearing races, cam lobes, etc., encountered in industrial applications.

A novel correction technique has been developed at Lambda Research for layer removal stress relaxation using finite element analysis (FEA). The technique allows separation of the geometric and stress distribution dependent portions of the generalized FEA correction. Therefore, an FEA correction developed for a given geometry and measurement direction, can be applied to any sample of the same shape containing different stress distributions. The new method allows deep measurement in complex components previously requiring neutron diffraction, yet avoids the problem of determining the unstressed lattice spacing.

(Continued on Page 2)

ANNOUNCEMENTS

Invitation to Tour Lambda Research

This fall the ASM/TMS Materials Exposition will be held in Cincinnati, October 8-10. We would like to extend an invitation to our clients who may be attending the conference to visit our facilities during their stay in Cincinnati. Located only 15 minutes from the Cincinnati Convention Center, this is an excellent opportunity to tour our laboratory facilities, and meet our technical staff. If you will be attending the conference, and would like to visit, please contact our Customer Service Representative, Kathleen Bauer, at (513) 561–0883 ext. 102, to schedule a convenient time and arrange for transportation.

Quantitative Analysis of Clays for Environmental Testing

Lambda Research has developed techniques and software for quantitative phase analysis for clay minerals. The procedure identifies and quantifies clays and other mineral species in soil samples to characterize the clay mineral components and the total clay fraction. The method was applied to characterize soils which may retain contaminated material in support of a major environmental study, and to quantify the non-carbonate portion of aggregates used in concrete.

Recently, we welcomed John Fox to our staff of Research Engineers. John received his Masters degree in Geology at the University of Illinois, and has more than 10 years of experience in mineralogy and petrographic analysis. John will be overseeing the qualitative and quantitative analysis testing services offered at Lambda Research. His analytical testing background with cement, clays, ceramics and minerals will be an asset to the testing capabilities and services we can offer our clients. John can be reached at (513) 561-0883 ext. 109.

The layer removal FEA correction technique involves creating a two or three dimensional model of the component from which layers of material are removed locally, forming a pocket. X-ray diffraction residual stress measurements are made in the layers successively exposed at the bottom of the pocket. The shape of the specimen and pocket are arbitrary. The choice of a two or three dimensional model depends upon the geometries of the specimen and electropolished pocket. A two dimensional model is adequate if plane stress, plane strain, or an axisymmetric stress state can be assumed. A three dimensional model is necessary in the general case.

The closed form solutions of Moore and Evans require the evaluation of an integral dependent upon both the residual stress distribution measured on each exposed layer, and the sample geometry. A similar approach for an arbitrary geometry and stress distribution would require prohibitively costly FEA solutions for each data set.

Fortunately, the stress distribution and geometry dependent portions of the FEA layer removal correction can be separated. The correction at any depth can be expressed as a summation over a series of geometric coefficients derived by FEA, indicating the fractional change in stress in the currently exposed layer caused by removal of the preceding layers. The sum of the coefficients weighted by the stress measured on each exposed layer down to the current depth gives the required stress correction. The matrix of coefficients depend only upon the sample geometry and direction of measurement.

The dependence of relaxation on geometry is fixed. Therefore, a correction matrix calculated for any fixed geometry, such as a gear tooth or turbine blade, can be applied to any residual stress field measured in the same direction, location, and geometry using the same layer removal pocket dimensions.

MATHEMATICAL BASIS AND FORMULATION

After each layer of elements are removed from the FE model, the magnitude of the stress redistribution in the direction of interest is calculated in the remaining levels. A relaxation coefficient is determined at each level which represents the amount of relaxation for that level as a result of removing layers above it. A lower triangular correction matrix is constructed which contains all of the coefficients for a given set of depths in a particular geometry, and which depends only on the geometry of the specimen.

The correction matrix contains the coefficients k_{ij}, derived from the FE model. The values of residual stress measured on the series of levels exposed by electropolishing are expressed in vector form. The product of the correction matrix and the measured residual stress values yields a vector of residual stress corrections,

$$\Delta\sigma_j = \sum_{j=1}^{i} K_{ij}\sigma_j \qquad 1$$

This vector of corrections is added to the measured residual stress vector to produce a final vector containing the residual stress distribution corrected properly for layer removal.

$$\sigma_{corrected} = \Delta\sigma + \sigma_{measured} \qquad 2$$

VERIFICATION OF THE FEA MATRIX RELAXATION CORRECTION TECHNIQUE

To demonstrate the validity of the FEA matrix method for layer removal correction, the FEA matrix solution was compared to the Moore and Evans closed form integral solutions for flat plate geometries. The accuracy of the numeric solutions was tested using idealized residual stress distributions generated in FEA models.

A synthetic residual stress distribution was applied throughout a two-dimensional FEA model of a flat plate. The residual stress distribution was designed to be a uniform stress field for each layer of elements, as required for the Moore and Evans solution.

The flat plate model was used to verify analytically that a matrix correction for a given geometry can be applied to an arbitrary stress field. The correction matrix used initially for a compressive distribution was further used to correct for layer removal with a tensile residual stress distribution in the same geometry.

The FEA matrix and Moore and Evans layer removal corrections for the flat plate are compared in Figure 1. The solutions are in excellent agreement, differing from the actual stress significantly only at the maximum depths. The partially relaxed stress which would be measured on the free surface exposed by electropolishing either a full layer, or a local 25 mm square pocket, show the magnitude of the potential error.

Residual Stress vs. Depth in Flat Plate

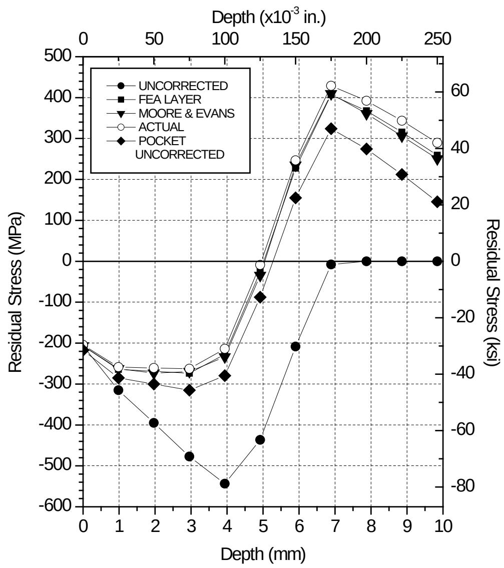

Fig. 1 - Comparison of correction methods and uncorrected results in a flat plate

APPLICATION TO COMPLEX GEOMETRIES

The primary benefit of the FEA matrix relaxation correction method is that it can be applied to the complex geometries of practical interest, where the assumptions of a simple geometry, uniform full layer removal, and symmetric stress fields are not valid. Gear teeth provide a typical example.

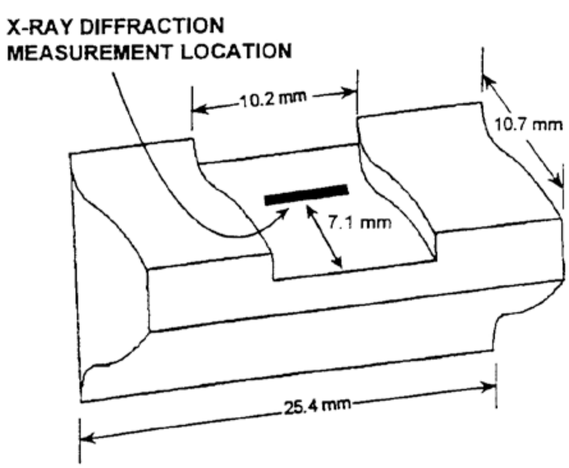

Fig. 2 - Three dimensional model of six-pitch gear tooth

An induction hardened SAE 1552 steel six-pitch spur gear was measured as a function of depth through the case at the pitch line of a tooth. Subsurface residual stress distributions are frequently measured at both pitch line and root-fillet radius locations, often through the case and into the softer core. To determine the relaxation resulting from layer removal at the pitch line location, a three dimensional finite element model was constructed, as shown in Figure 2. The residual stress was measured in the direction from root-to-crown at various depths into the tooth using a 1 mm by 5 mm irradiated area.

The subsurface residual stress distribution measured in the induction hardened gear is shown in Figure 3 without correction, and with both the flat plate approximation and the FEA matrix corrections. With the proper FEA correction, the residual stress distribution crosses into tension at approximately 1.0 mm, the depth of the case-core interface. The uncorrected distribution not only fails to accurately reveal the depth of the compressive case, but does not indicate equilibrium.

RADIAL RESIDUAL STRESS AT PITCHLINE OF GEAR TOOTH

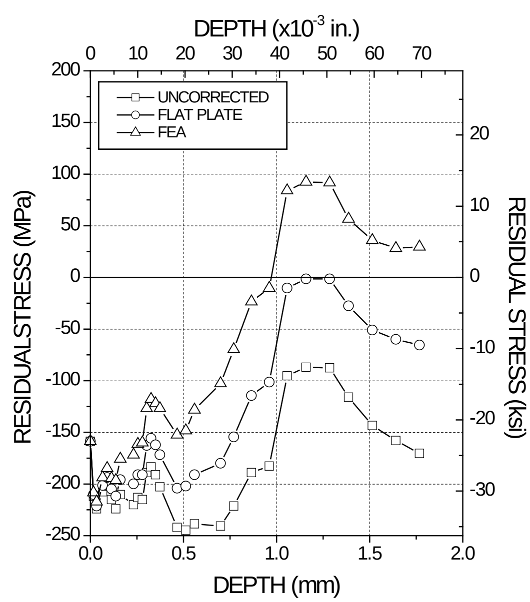

Fig. 3 - Comparison of uncorrected and corrected gear tooth and flat plate

CONCLUSION

The FEA matrix layer removal correction method allows residual stress distributions to be measured in complex geometries having arbitrary stress distributions to depths which were previously unobtainable. X-ray diffraction residual stress measurement can now be used in many applications which previously required neutron diffraction. Once the FEA matrix has been developed, it can be applied to any component of the same geometry, lending itself to quality control testing of gear teeth, bearing races, turbine blades, etc.

REFERENCES

1. Prevey, P.S., "Problems with Non-Destructive Surface Residual Stress Measurement," Practical Applications of Residual Stress Technology, ed. C. Ruud, Materials Park, OH: American Society for Metals, 1991, pp. 47-54.

2. Moore, M.G. and Evans, W.P., "Mathematical Correction for Stress in Removed Layers in X-Ray Diffraction Residual Stress Analysis," SAE Transactions, Vol. 66, 1958, pp. 340-345.